LESSON PLAN – Dr. of Yo-yo-ology

Academic Subject: Physics, physical science

Academic Topics: Moment of inertia, rotational kinetic energy, conservation of energy

Background: The yo-yo has been around for a very long time. In fact, it is considered one of the oldest toys in history – second only to the doll. Today's yo-yo is much different from yo-yos of the past. Up until fairly recently, getting a yo-yo to sleep for 20 seconds was considered very impressive. With vast improvements in yo-yo design, it is now possible to make yo-yos sleep for much longer periods of time. At the 2004 World Yo-Yo Championships Rick Wyatt made his yo-yo sleep for 14.03 minutes – a new record.

Discussion: A yo-yo is said to be sleeping when it is spinning at the end of the string. A yo-yo that can sleep longer can do more tricks, and tricks are what yo-yoing is all about. Therefore the goal when designing a yo-yo is to create one that has a tendency to keep on spinning.

Inertia is a concept familiar to most physics students – an object's resistance to a change in state. A truck, with its high inertia (large mass), is difficult to get moving from rest and also hard to stop once it is moving. When examining the yo-yo we must consider its moment of inertia – the higher this is, the greater its tendency to keep on spinning. An object's moment of inertia depends on how its mass is distributed relative to its axis of rotation. Different shapes have different moments of inertia. Since a yo-yo is approximately a solid cylinder spinning about an axis through its center we can use the following formula to find its moment of inertia:

$$\boxed{I = \frac{1}{2}MR^2}$$

where M is the mass and R is the radius of the cylinder.

The more mass away from the axis of rotation, the greater the yoyo's moment of inertia. For this reason, many people add weight to the outer rims of the yo-yo. Although this means that it will be harder to get the yo-yo spinning initially, once it is spinning it will be harder to stop.

With increased sleep time comes a whole litany of tricks that can be done with the yo-yo, but the simplest of all is to tug on the rope and make the yo-yo come back up. Yo-yos are known for their rises and falls – so much so that a person whose weight fluctuates up and down is said to be a yo-yo dieter. Let's examine the energy transformations that take place during the fall and rise of the yo-yo.

Before the yo-yo is dropped it has only gravitational potential energy. When released the yo-yo begins falling and spinning. Therefore it gains (linear) kinetic energy and also rotational kinetic energy.

$\boxed{KE_{rotational} = \frac{1}{2} I\omega^2}$ where I is moment of inertia and ω is angular velocity.

Angular velocity is the number of radians the yo-yo rotates through each second and is related to linear velocity using the following relationship: $\boxed{v = \omega r}$

If friction is negligible, we can say the following about the yo-yo's energy:

$$\boxed{\begin{aligned} &Energy_{Top} = Energy_{Halfway} = Energy_{Bottom} \\ &GPE = GPE + KE + KE_{rot} = KE_{rot} \end{aligned}}$$

Extending the Lesson:
Ask for a volunteer to hold a meter stick in the center and attach a weight to the meter stick on each side of the volunteer's hand. Have the volunteer spin the meter stick. Now slide the weights to the end of the meter stick and have the volunteer again try to twirl the meter stick. It should be much harder to get it going. Although the meter stick had the same amount of mass in both cases, it had more rotational inertia when the weights were farther from the axis of rotation.

ACTIVITY SHEET

1. Determine the moment of inertia of a yo-yo with a mass of 150 grams and a radius of 1.5 cm.

2. The mass of a yo-yo is 200 grams and its radius is 1.7 cm. How much rotational kinetic energy does a sleeping yo-yo have it makes 2 revolutions each second?

3. A yo-yo of radius 1.5 cm is wound and dropped from a height of 0.8 m. Ignoring friction, how many revolutions does it make each second when it is sleeping at the lowest point?

4. The yo-yo in the previous problem is given a tug and begins to rise. How fast is it moving when it is 0.4 m above its lowest point?

ANSWERS

1. $I = \frac{1}{2}MR^2$

$I = \frac{1}{2}(.15kg)(.015m)^2$

$I = \boxed{1.7x10^{-5}\,kg * m^2}$

2. $\omega = 2\frac{rev}{s} * \frac{2\pi rad}{1rev} = 4\pi\frac{rad}{s}$

$KE_{rot} = \frac{1}{2}I\omega^2$

$KE_{rot} = \frac{1}{2}(\frac{1}{2} * .2kg * (.017m)^2)(4\pi\frac{rad}{s})^2$

$KE_{rot} = \boxed{2.3x10^{-3}\,J}$

3. Set total energy at the top = to total energy at the bottom.

$GPE = KE_{rot}$

$\cancel{m}gh = \frac{1}{2}(\frac{1}{2}\cancel{m}r^2)\omega^2$

$9.8\frac{m}{s^2}(.8m) = \frac{1}{4}(.015m)^2\omega^2$

$\omega = 373\frac{rad}{s}$

$373\frac{rad}{s} * \frac{1rev}{2\pi rad} = \boxed{60\frac{rev}{s}}$

4. Set total energy at the top = to total energy at the midpoint. Yo-yo will only fall 0.4 m.

$GPE = KE + KE_{rot}$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}(\frac{1}{2}\cancel{m}r^2)\omega^2$

$gh = \frac{1}{2}v^2 + \frac{1}{4}(r^2)(\frac{v}{r})^2$

$gh = \frac{1}{2}v^2 + \frac{1}{4}(v^2)$

$9.8\frac{m}{s^2}(0.4m) = \frac{3}{4}v^2$

$\boxed{v = 2.3m/s}$